
dolphin_navier_scipy Documentation

Release 1.0

highlando

May 04, 2017

Contents

1	Code Reference	3
1.1	dolfin_to_sparrays	3
1.2	stokes_navier_utils	7
1.3	problem_setups	11
1.4	data_output_utils	14
2	Indices and tables	15
	Python Module Index	17

The package *dolfin_navier_scipy* (*dns*) provides an interface between *scipy* and *FEniCS* in view of solving Navier-Stokes Equations. *FEniCS* is used to perform a Finite Element discretization of the equations. The assembled coefficients are exported as sparse matrices for use in *scipy*. Nonlinear and time-dependent parts are evaluated and assembled on demand. Visualization is done via the *FEniCS* interface to *paraview*.

Contents:

dolfin_to_sparrays

`dolfin_to_sparrays.ass_convmat_asmatquad` (*W=None, invindsw=None*)

assemble the convection matrix H, so that $N(v)v = H[v.v]$

for the inner nodes.

Notes

Implemented only for 2D problems

`dolfin_to_sparrays.get_stokessysmats` (*V, Q, nu=None, bccontrol=False, cbclist=None, cb-shapefuncs=None*)

Assembles the system matrices for Stokes equation

in mixed FEM formulation, namely

$$\begin{bmatrix} A & -J' \\ J & 0 \end{bmatrix} : V \times Q \rightarrow V' \times Q'$$

as the discrete representation of

$$\begin{bmatrix} -\Delta & \text{grad} \\ \text{div} & 0 \end{bmatrix}$$

plus the velocity and pressure mass matrices

for a given trial and test space $W = V * Q$ not considering boundary conditions.

Parameters **V** : `dolfin.VectorFunctionSpace`

Fenics `VectorFunctionSpace` for the velocity

Q : `dolfin.FunctionSpace`

Fenics `FunctionSpace` for the pressure

nu : float, optional

viscosity parameter - defaults to 1

bcecontrol : boolean, optional

whether boundary control (via penalized Robin conditions) is applied, defaults to *False*

cbclist : list, optional

list of dolfin's Subdomain classes describing the control boundaries

cbshapefuns : list, optional

list of spatial shape functions of the control boundaries

Returns **stokesmats**, dictionary :

a dictionary with the following keys:

- **M**: the mass matrix of the velocity space,
- **A**: the stiffness matrix $\nu \int_{\Omega} (\nabla \phi_i, \nabla \phi_j)$
- **JT**: the gradient matrix,
- **J**: the divergence matrix, and
- **MP**: the mass matrix of the pressure space
- **Apbc**: (N, N) sparse matrix, the contribution of the Robin conditions to A
 $\nu \int_{\Gamma} (\phi_i, \phi_j)$
- **Bpbc**: (N, k) sparse matrix, the input matrix of the Robin conditions $\nu \int_{\Gamma} (\phi_i, g_k)$,
where g_k is the shape function associated with the j-th control boundary segment

`dolphin_to_sparrays.get_convmts` (*u0_dolfun=None, u0_vec=None, V=None, invinds=None, diribcs=None*)

returns the matrices related to the linearized convection

where *u_0* is the linearization point

Returns **N1** : (N,N) sparse matrix

representing $(u_0 \cdot \nabla)u$

N2 : (N,N) sparse matrix

representing $(u \cdot \nabla)u_0$

fv : (N,1) array

representing $(u_0 \cdot \nabla)u_0$

See also:

`stokes_navier_utils.get_v_conv_conts` the convection contributions reduced to the inner nodes

`dolphin_to_sparrays.get_curfv` (*V, fv, invinds, tcur*)

get the fv at innernodes at t=tcur

`dolphin_to_sparrays.get_convvec` (*u0_dolfun=None, V=None, u0_vec=None, femp=None, diribcs=None, invinds=None*)

return the convection vector e.g. for explicit schemes

given a dolfin function or the coefficient vector

`dolphin_to_spararrays.condense_sysmatsbybcs` (*stms*, *velbcs*)
resolve the Dirichlet BCs and condense the system matrices
to the inner nodes

Parameters *stms*: dict :

of the stokes matrices with the keys

- *M*: the mass matrix of the velocity space,
- *A*: the stiffness matrix,
- *JT*: the gradient matrix,
- *J*: the divergence matrix, and
- *MP*: the mass matrix of the pressure space

velbcs : list

of dolfin Dirichlet boundary conditions for the velocity

Returns *stokesmatsc* : dict

a dictionary of the condensed matrices:

- *M*: the mass matrix of the velocity space,
- *A*: the stiffness matrix,
- *JT*: the gradient matrix, and
- *J*: the divergence matrix
- *MP*: the mass matrix of the pressure space

rhsvecsb : dict

a dictionary of the contributions of the boundary data to the rhs:

- *f_v*: contribution to momentum equation,
- *f_p*: contribution to continuity equation

invinds : (N,) array

vector of indices of the inner nodes

bcinds : (K,) array

vector of indices of the boundary nodes

bivals : (K,) array

vector of the values of the boundary nodes

`dolphin_to_spararrays.condense_velmatsbybcs` (*A*, *velbcs*, *return_bcinfo=False*)
resolve the Dirichlet BCs, condense velocity related matrices

to the inner nodes, and compute the rhs contribution This is necessary when, e.g., the convection matrix changes with time

Parameters *A* : (N,N) sparse matrix

coefficient matrix for the velocity

velbcs : list

of dolfin *dolphin* Dirichlet boundary conditions for the velocity

return_binfo : boolean, optional

if *True* a dict with the inner and the boundary indices is returned, defaults to *False*

Returns **Ac** : (K, K) sparse matrix

the condensed velocity matrix

fvbc : (K, 1) array

the contribution to the rhs of the momentum equation

dict, on demand :

with the keys

- **ininds**: indices of the inner nodes
- **bcinds**: indices of the boundary nodes

`dolphin_to_spararrays.expand_vp_dolfunc` (*V=None, Q=None, invinds=None, diribcs=None, vp=None, vc=None, pc=None, ppin=-1, **kwargs*)

expand *v* [and *p*] to the dolfin function representation

Parameters **V** : dolfin.VectorFunctionSpace

FEM space of the velocity

Q : dolfin.FunctionSpace

FEM space of the pressure

invinds : (N,) array

vector of indices of the velocity nodes

diribcs : list, optional

of the (Dirichlet) velocity boundary conditions, if *None* it is assumed that *vc* already contains the bc, defaults to *None*

vp : (N+M,1) array, optional

solution vector of velocity and pressure

vc : (N,1) array, optional

solution vector of velocity

pc : (M,1) array, optional

solution vector of pressure

ppin : {int, None}, optional

which dof of *p* is used to pin the pressure, defaults to *-1*

Returns **v** : dolfin.Function(V)

velocity as function

p : dolfin.Function(Q), optional

pressure as function

See also:

[`expand_vecnbc_dolfunc`](#) for a scalar function with multiple bcs

`dofin_to_spararrays.expand_vecnbc_dolfunc` (*V=None, vec=None, bcinds=None, bcvals=None, diribcs=None, bcsfaclist=None, invinds=None*)

expand a function vector with changing boundary conditions

the boundary conditions may not be disjoint, what is used to model spatial dependencies of a control at the boundary.

Parameters **V** : `dofin.FunctionSpace`

FEM space of the scalar

invinds : (N,) array

vector of indices of the velocity nodes

vec : (N,1) array

solution vector

diribcs : list

of boundary conditions

bcsfaclist : list, optional

of factors for the boundary conditions

Returns `dofin.function` :

of the vector values and the bcs

`dofin_to_spararrays.append_bcs_vec` (*vvec, V=None, vdim=None, invinds=None, diribcs=None, **kwargs*)

append given boundary conditions to a vector representing inner nodes

`dofin_to_spararrays.mat_dofin2sparse` (*A*)

get the csr matrix representing an assembled linear dof in form

stokes_navier_utils

`stokes_navier_utils.get_v_conv_conts` (*prev_v=None, V=None, invinds=None, diribcs=None, Picard=False*)

get and condense the linearized convection

to be used in a Newton scheme

$$(u \cdot \nabla)u \rightarrow (u_0 \cdot \nabla)u + (u \cdot \nabla)u_0 - (u_0 \cdot \nabla)u_0$$

or in a Picard scheme

$$(u \cdot \nabla)u \rightarrow (u_0 \cdot \nabla)u$$

Parameters **prev_v** : (N,1) ndarray

convection velocity

V : `dofin.VectorFunctionSpace`

FEM space of the velocity

invinds : (N,) ndarray or list

indices of the inner nodes

diribcs : list

of dofing Dirichlet boundary conditons

Picard : boolean

whether Picard linearization is applied, defaults to *False*

Returns convc_mat : (N,N) sparse matrix

representing the linearized convection at the inner nodes

rhs_con : (N,1) array

representing $(u_0 \cdot \nabla)u_0$ at the inner nodes

rhsv_conbc : (N,1) ndarray

representing the boundary conditions

`stokes_navier_utils.solve_nse` (*A=None, M=None, J=None, JT=None, fv=None, fp=None, fvc=None, fpc=None, fv_tmdp=None, fv_tmdp_params={}, fv_tmdp_memory=None, iniv=None, lin_vel_point=None, stokes_flow=False, trange=None, t0=None, tE=None, Nts=None, V=None, Q=None, invinds=None, diribcs=None, output_includes_bcs=False, N=None, nu=None, ppin=-1, closed_loop=False, static_feedback=False, feedbackthroughdict=None, return_vp=False, tb_mat=None, c_mat=None, vel_nwtm_stps=20, vel_nwtm_tol=5e-15, vel_pcrd_stps=4, krylov=None, krpslvprms={}, krplsprms={}, clearprvdata=False, get_datastring=None, data_prfx='', paraviewoutput=False, vfileprfx='', pfileprfx='', return_dictofvelstrs=False, return_dictofpstrs=False, dictkeysstr=False, comp_nonl_semexp=False, return_as_list=False, start_sstokes=False, **kw*)

solution of the time-dependent nonlinear Navier-Stokes equation

$$\begin{aligned} M\dot{v} + Av + N(v)v + J^T p &= f \\ Jv &= g \end{aligned}$$

using a Newton scheme in function space, i.e. given v_k , we solve for the update like

$$M\dot{v} + Av + N(v_k)v + N(v)v_k + J^T p = N(v_k)v_k + f,$$

and trapezoidal rule in time. To solve an *Oseen* system (linearization about a steady state) or a *Stokes* system, set the number of Newton steps to one and provide a linearization point and an initial value.

Parameters lin_vel_point : dictionary, optional

contains the linearization point for the first Newton iteration

- Steady State: $\{\{None: \text{'path_to_narray'}\}, \{None: \text{narray}\}\}$
- Newton: $\{t: \text{'path_to_narray'}\}$

defaults to *None*

dictkeysstr : boolean, optional

whether the *keys* of the result dictionaries are strings instead of floats, defaults to *False*

fv_tmdp : callable $f(t, v, \text{dict})$, optional

time-dependent part of the right-hand side, set to zero if *None*

fv_tmdp_params : dictionary, optional

dictionary of parameters to be passed to *fv_tmdp*, defaults to *{}*

fv_tmdp_memory : dictionary, optional

memory of the function

output_includes_bcs : boolean, optional

whether append the boundary nodes to the computed and stored velocities, defaults to *False*

krylov : {None, 'gmres'}, optional

whether or not to use an iterative solver, defaults to *None*

krpssprms : dictionary, optional

to specify parameters of the linear solver for use in Krypy, e.g.,

- initial guess
- tolerance
- number of iterations

defaults to *None*

krplsprms : dictionary, optional

parameters to define the linear system like

- preconditioner

ppin : {int, None}, optional

which dof of *p* is used to pin the pressure, defaults to *-1*

stokes_flow : boolean, optional

whether to consider the Stokes linearization, defaults to *False*

start_ssstokes : boolean, optional

for your convenience, compute and use the steady state stokes solution as initial value, defaults to *False*

Returns dictofvelstrs : dictionary, on demand

dictionary with time *t* as keys and path to velocity files as values

dictofpstrs : dictionary, on demand

dictionary with time *t* as keys and path to pressure files as values

vellist : list, on demand

list of the velocity solutions

```
stokes_navier_utils.solve_steadystate_nse(A=None, J=None, JT=None, M=None,
                                           fv=None, fp=None, V=None, Q=None,
                                           invinds=None, diribcs=None, re-
                                           turn_vp=False, ppin=-1, N=None,
                                           nu=None, vel_pcrd_stps=10,
                                           vel_pcrd_tol=0.0001, vel_nwtm_stps=20,
                                           vel_nwtm_tol=5e-15, clearprvdata=False,
                                           vel_start_nwtm=None, get_datastring=None,
                                           data_prfx='', paraviewoutput=False,
                                           save_intermediate_steps=False, vfileprfx='',
                                           pfileprfx='', **kw)
```

Solution of the steady state nonlinear NSE Problem

using Newton's scheme. If no starting value is provided, the iteration is started with the steady state Stokes solution.

Parameters **A** : (N,N) sparse matrix

stiffness matrix aka discrete Laplacian, note the sign!

M : (N,N) sparse matrix

mass matrix

J : (M,N) sparse matrix

discrete divergence operator

JT : (N,M) sparse matrix, optional

discrete gradient operator, set to J.T if not provided

fv, fp : (N,1), (M,1) ndarrays

right hand sides restricted via removing the boundary nodes in the momentum and the pressure freedom in the continuity equation

ppin : {int, None}, optional

which dof of p is used to pin the pressure, defaults to -1

return_vp : boolean, optional

whether to return also the pressure, defaults to *False*

vel_pcrd_stps : int, optional

Number of Picard iterations when computing a starting value for the Newton scheme, cf. Elman, Silvester, Wathen: *FEM and fast iterative solvers*, 2005, defaults to 100

vel_pcrd_tol : real, optional

tolerance for the size of the Picard update, defaults to $1e-4$

vel_nwtm_stps : int, optional

Number of Newton iterations, defaults to 20

vel_nwtm_tol : real, optional

tolerance for the size of the Newton update, defaults to $5e-15$

```
stokes_navier_utils.get_pfromv(v=None, V=None, M=None, A=None, J=None, fv=None,
                                fp=None, diribcs=None, invinds=None, **kwargs)
```

for a velocity v , get the corresponding p

Notes

Formula is only valid for constant rhs in the continuity equation

problem_setups

`problem_setups.get_sysmats` (*problem='drivencavity', N=10, scheme=None, ppin=None, Re=None, nu=None, bccontrol=False, mergerhs=False, onlymesh=False*)
retrieve the system matrices for stokes flow

Parameters **problem** : {'drivencavity', 'cylinderwake'}

problem class

N : int

mesh parameter

nu : real, optional

kinematic viscosity, is set to L/Re if Re is provided

Re : real, optional

Reynoldsnumber, is set to L/nu if nu is provided

bccontrol : boolean, optional

whether to consider boundary control via penalized Robin defaults to *False*

mergerhs : boolean, optional

whether to merge the actual rhs and the contribution from the boundary conditions into one rhs

onlymesh : boolean, optional

whether to only return *femp*, containing the mesh and FEM spaces, defaults to *False*

Returns **femp** : dict

with the keys:

- *V*: FEM space of the velocity
- *Q*: FEM space of the pressure
- *diribcs*: list of the (Dirichlet) boundary conditions
- *bcinds*: indices of the boundary nodes
- *bcvals*: values of the boundary nodes
- *invinds*: indices of the inner nodes
- *fv*: right hand side of the momentum equation
- *fp*: right hand side of the continuity equation
- *charlen*: characteristic length of the setup
- *nu*: the kinematic viscosity
- *Re*: the Reynolds number
- *odcoo*: dictionary with the coordinates of the domain of observation

- *cdcoo*: dictionary with the coordinates of the domain of **ppin* [{int, None}]

which dof of *p* is used to pin the pressure, typically *-1* for internal flows, and *None* for flows with outflow control

stokesmatssc : dict

a dictionary of the condensed matrices:

- *M*: the mass matrix of the velocity space,
- *A*: the stiffness matrix,
- *JT*: the gradient matrix, and
- *J*: the divergence matrix
- *Jfull*: the uncondensed divergence matrix

and, if *bccontrol=True*, the boundary control matrices that weakly impose $Arob*v = Brob*u$, where

- *Arob*: contribution to *A*
- *Brob*: input operator

‘if mergerhs’ :

rhsd : dict

rhsd_vfrc and *rhsd_stbc* merged

‘else’ :

rhsd_vfrc : dict

of the dirichlet and pressure fix reduced right hand sides

rhsd_stbc : dict

of the contributions of the boundary data to the rhs:

- *fv*: contribution to momentum equation,
- *fp*: contribution to continuity equation

Examples

```
femp, stokesmatssc, rhsd_vfrc, rhsd_stbc = get_sysmats(problem='drivencavity', N=10, nu=1e-2)
```

```
problem_setups.drivcav_fems (N, vdgree=2, pdgree=1, scheme=None, bccontrol=None)
```

dictionary for the fem items of the (unit) driven cavity

Parameters *N* : int

mesh parameter for the unitsquare (*N* gives $2*N*N$ triangles)

vdgree : int, optional

polynomial degree of the velocity basis functions, defaults to 2

pdgree : int, optional

polynomial degree of the pressure basis functions, defaults to 1

scheme : {None, ‘CR’, ‘TH’ }

the finite element scheme to be applied, ‘CR’ for Crouzieux-Raviart, ‘TH’ for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*

bccontrol : boolean, optional

whether to consider boundary control via penalized Robin defaults to false.
 TODO: not implemented yet but we need it here for consistency

Returns femp : a dict

of problem FEM description with the keys:

- *V*: FEM space of the velocity
- *Q*: FEM space of the pressure
- *diribcs*: list of the (Dirichlet) boundary conditions
- *fv*: right hand side of the momentum equation
- *fp*: right hand side of the continuity equation
- *charlen*: characteristic length of the setup
- *odcoo*: dictionary with the coordinates of the domain of observation
- *cdcoo*: dictionary with the coordinates of the domain of control

`problem_setups.cyl_fems (refinement_level=2, vdgree=2, pdgree=1, scheme=None, bccontrol=False, verbose=False)`
 dictionary for the fem items for the cylinder wake

Parameters N : mesh parameter for the unitsquare (N gives $2*N*N$ triangles)

vdgree : polynomial degree of the velocity basis functions,
 defaults to 2

pdgree : polynomial degree of the pressure basis functions,
 defaults to 1

scheme : {None, ‘CR’, ‘TH’}

the finite element scheme to be applied, ‘CR’ for Crouzieux-Raviart, ‘TH’ for Taylor-Hood, overrides *pdgree*, *vdgree*, defaults to *None*

bccontrol : boolean, optional

whether to consider boundary control via penalized Robin defaults to *False*

Returns femp : a dictionary with the keys:

- *V*: FEM space of the velocity
- *Q*: FEM space of the pressure
- *diribcs*: list of the (Dirichlet) boundary conditions
- *dirip*: list of the (Dirichlet) boundary conditions for the pressure
- *fv*: right hand side of the momentum equation
- *fp*: right hand side of the continuity equation
- *charlen*: characteristic length of the setup
- *odcoo*: dictionary with the coordinates of the domain of observation
- *cdcoo*: dictionary with the coordinates of the domain of control

- *uspacedep*: int that specifies in what spatial direction Bu changes. The remaining is constant
- *bcsubdoms*: list of subdomains that define the segments where the boundary control is applied

Notes

parts of the code were taken from the NSbench collection <https://launchpad.net/nsbench>

```
__author__ = "Kristian Valen-Sendstad <kvs@simula.no>"
__date__ = "2009-10-01"
__copyright__ = "Copyright (C) 2009-2010 " + __author__
__license__ = "GNU GPL version 3 or any later version"
```

data_output_utils

```
data_output_utils.output_paraview(V=None, Q=None, fstring='nn', invinds=None,
                                   diribcs=None, vp=None, vc=None, pc=None, ppin=-1,
                                   t=None, writeoutput=True, vfile=None, pfile=None)
```

write the paraview output for a solution vector vp

```
data_output_utils.load_or_comp(filestr=None, comprtn=None, comprtnargs={}, array-
                                type=None, debug=False, loadrtn=None, loadmsg='loaded',
                                savetrn=None, savemsg='saved', itsadict=False,
                                numthings=1)
```

routine for caching computation results on disc

Parameters *filestr*: {string, list of strings, 'None'} :

where to load/store the computed things, if *None* nothing is loaded or stored

arraytype: {'None', 'sparse', 'dense'} :

if not *None*, then it sets the default routines to save/load dense or sparse arrays

itsadict: boolean, optional :

whether it is *python dictionary* that can be JSON serialized, overrides all other options concerning arrays

savetrn: fun(), optional :

routine for saving the computed results, defaults to *None*, i.e. no saving here

debug: boolean, optional :

no saving or loading, defaults to *False*

CHAPTER 2

Indices and tables

- `genindex`
- `modindex`
- `search`

d

`data_output_utils`, [14](#)
`dolfin_to_spararrays`, [3](#)

p

`problem_setups`, [11](#)

s

`stokes_navier_utils`, [7](#)

A

append_bcs_vec() (in module dolfin_to_spararrays), 7
ass_convmat_asmatquad() (in module
dolfin_to_spararrays), 3

C

condense_sysmatsbybcs() (in module
dolfin_to_spararrays), 4
condense_velmatsbybcs() (in module
dolfin_to_spararrays), 5
cyl_fems() (in module problem_setups), 13

D

data_output_utils (module), 14
dolfin_to_spararrays (module), 3
drivcav_fems() (in module problem_setups), 12

E

expand_vecnbc_dolfunc() (in module
dolfin_to_spararrays), 6
expand_vp_dolfunc() (in module dolfin_to_spararrays), 6

G

get_convsmats() (in module dolfin_to_spararrays), 4
get_convvec() (in module dolfin_to_spararrays), 4
get_curfv() (in module dolfin_to_spararrays), 4
get_pfromv() (in module stokes_navier_utils), 10
get_stokessysmats() (in module dolfin_to_spararrays), 3
get_sysmats() (in module problem_setups), 11
get_v_conv_confs() (in module stokes_navier_utils), 7

L

load_or_comp() (in module data_output_utils), 14

M

mat_dolfin2sparse() (in module dolfin_to_spararrays), 7

O

output_paraview() (in module data_output_utils), 14

P

problem_setups (module), 11

S

solve_nse() (in module stokes_navier_utils), 8
solve_steadystate_nse() (in module stokes_navier_utils),
9
stokes_navier_utils (module), 7